The Ultra Wideband Transfer Function Representation of Complex Three-Dimensional Electromagnetic Structures

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Abstract — the network-oriented ultra wideband transfer function representation of complex three-dimensional electromagnetic structures is investigated. The transfer function is consisting of two parts: entire function and rational or pole function. The reduction of a rank of an ill-conditioned matrix is performed in accordance with the spectral criterion. System identification is carried out by the Matrix Pencil Method (MPM) and a criterion for the model order selection is introduced. The presented method yields considerable reduction of computational effort and allows to generate compact models of electromagnetic systems.

***Index Terms* — Modeling, Frequency-domain synthesis, Transfer functions, Time-domain synthesis.**

I. INTRODUCTION

The need of novel and broadband circuitry in nowadays applications, especially in mobile communication and vehicular sensing equipments, demands the availability of reliable and accurate modelling of complex electromagnetic structures up to the millimetre wave range. The simulation of complex three-dimensional electromagnetic structures is resulting in an enormous computational effort and correspondingly large times for simulation. So the application of network-oriented modelling [1] in connection with system identification techniques allows reducing the computational effort and the computational time by orders of magnitude.

In network theory systematic approaches for one-port or multiport circuit analysis are based on the transfer function or impulse response representation of the circuit elements. This representation may be obtained through pole extraction from the numerical solution of the field problem for modelling electromagnetic structures.

The transmission line matrix (TLM) method [2] or finite-difference time-domain (FDTD) method [3] allows to provide the full-wave analysis of complex three-dimensional electromagnetic structures made of arbitrary materials. This is extremely important for ultra wideband (UWB) electromagnetic structures, because time-domain simulation allows to characterize the properties of these structures in a wide frequency band by computing the response to a single impulsive excitation.

According to the singularity expansion method [4] the signal, scattered by the electromagnetic structure, can be decomposed into early-time and late-time parts. The early-time reaction is limited in time-domain and consists of the forced oscillations. This part can be represented by the entire function, which depends on the driving signal. The late-time response contains only natural oscillations and can be defined by the parameters of the electromagnetic structure.

The subdivision of the total reaction into two parts and modeling of the structure by using only the late-time response leads to an incorrect transfer function representation of the electromagnetic structure. First of all natural oscillations occur not only in the late-time response, but in the early-time part of the reaction too. To use this part of the total reaction for the network-oriented transfer function representation forced oscillations must be removed from the signal, scattered by the electromagnetic structure. It can be done by computing the ratio of the corresponding Fourier transforms of the time-domain waveforms of the scattered and input signals in frequency domain or by deconvolution of the scattered signal in time-domain. Moreover, the main portion of the total signal energy concentrates in the early-time reaction. The late-time response contains relatively low energy and so becomes substantially corrupted with noise.

The deconvolution in time-domain can be done by the matrix inversion, but the matrix obtained in this way is ill-conditioned. The reduction of a rank of an ill-conditioned matrix is performed in accordance with the spectral criterion [7]. This criterion needs to take into consideration only the spectral components of the driving signal, exceeding some threshold within the frequency bandwidth of the electromagnetic structure.

The application of MPM to the estimation of transfer functions of complex 3-dimentional structures requires a strategy of choosing the order of the method. This strategy uses the analysis of the spectrum of the difference signal between original and reconstructed system impulse response. Another problem is the influence of the entire function on the accuracy of poles estimation. The method is applied to the electromagnetic full-wave simulation of a patch antenna. For the electromagnetic simulation the TLM method is used [2]. Computational results with and without applying system identification are compared with measured data.

II. THEORY OF TRANSFER FUNCTION REPRESENTATION

It is known, that lossless reciprocal three-dimensional electromagnetic structures can be represented by the dyadic Green's function [5]. For example, the Green's functions in admittance representation is given by

$$\underline{\underline{Y}}(\mathbf{x},\mathbf{x}',\omega)=\frac{1}{j\omega}\underline{\underline{y}}^{0}(\mathbf{x},\mathbf{x}')+ \\ +\sum_{\lambda}\frac{1}{j\omega}\frac{\omega^{2}}{\omega^{2}-\omega_{\lambda}^{2}}\underline{\underline{y}}^{\lambda}(\mathbf{x},\mathbf{x}'). \quad (1)$$

The dyadic $\underline{\underline{y}}^{0}(\mathbf{x},\mathbf{x}')$ represent the static part of the Green's function, whereas term $\underline{\underline{y}}^{\lambda}(\mathbf{x},\mathbf{x}')$ corresponds to a pole at the frequency ω_λ. Electric and magnetic fields may be expanded into a series of basis field functions, the amplitudes of which can be treated as generalized voltages and generalized currents respectively.

In general case, when we have an excitation pulse with finite bandwidth $X_{ex}(f)$ and receive a scattered signal with spectrum $X_{sc}(f)$, which represents electric or magnetic component of the scattered field, according to the singularity expansion method the equation for the spectrum of the scattered signal is following [4]:

$$X_{sc}(f)=X^{ENT}(f)+\sum_{n=1}^{N}\frac{A_n}{j2\pi f-s_n}, \quad (2)$$

where $X^{ENT}(f)$ is described by the entire function, the sum represents the pure resonant portion of the scattered signal, N is a number of poles $s_n=-\sigma_n+j\omega_n$ on the complex s-plane, A_n is the residues of corresponding poles.

To remove the excitation signal $X_{ex}(f)$ from the scattered field expressions we can compute the ratio of the spectra of scattered signal and driving signal. This allows to obtain the transfer function representation of the system

$$H(f)=\frac{X_{sc}(f)}{X_{ex}(f)}=H^{ENT}(f)+\sum_{n=1}^{N}\frac{B_n}{j2\pi f-s_n}, \quad (3)$$

where $H^{ENT}(f)$ is the entire part of transfer function.

Time-domain representation of the system is given by the impulse response, which is the inverse Fourier transform of the transfer function (3)

$$h(t)=h^{ENT}(t)+2\sum_{n=1}^{N/2}|B_n|\exp(-\sigma_n t)\cos(\omega_n t+\varphi_n). \quad (4)$$

Deconvolution of the received signal is done in time-domain. We sample the scattered signal

$$x_{\Delta}^{SC}(t)=T\sum_{m=0}^{M-1}x_m^{SC}\delta(t-mTs), \quad (5)$$

where Ts is a sampling period and M is the number of samples. The scattered signal can be represented by linear convolution of the excitation signal and the impulse response

$$x_m^{SC}=\sum_{k=0}^{M-1}x_k^{EX}h_{m-k}, \quad (6)$$

where h_m and x_m^{EX} are the samples of impulse response and excitation signal respectively. We can express (6) in matrix form

$$\begin{bmatrix} x_0^{SC} \\ \ldots \\ x_{M-1}^{SC}\end{bmatrix}=\begin{bmatrix} x_0^{EX} & \ldots & 0 \\ \ldots & \ldots & \ldots \\ x_{M-1}^{EX} & \ldots & x_0^{EX}\end{bmatrix}\cdot\begin{bmatrix} h_0 \\ \ldots \\ h_{M-1}\end{bmatrix}. \quad (7)$$

Equation (7) can be solved for **h** by inversion of the excitation matrix $\mathbf{x}^{EX}$. The main problem is that this matrix is ill-conditioned.

The condition number Q_s of the excitation matrix $\mathbf{x}^{EX}$ gives a measure of the accuracy of the computation. It relates relative variations of $\mathbf{x}^{SC}$ with relative variations of **h**:

$$\frac{\|\Delta\mathbf{h}\|}{\|\mathbf{h}\|}\le Q_{\mathbf{s}}\frac{\|\Delta\mathbf{x}^{SK}\|}{\|\mathbf{x}^{SK}\|}, \quad (8)$$

where $\Delta\mathbf{h}$ and $\Delta\mathbf{x}^{SK}$ are absolute deviations of the impulse response **h** and scattered signal $\mathbf{x}^{SK}$ respectively, the symbol || • || denotes the root-mean-square value or the norm of the corresponding vector. For large Q_s the matrix $\mathbf{x}^{EX}$ is ill-conditioned and even slight variations of the scattered vector can cause very large errors in the estimated impulse response h(t).

There are many different explanations of the reasons for matrix ill-conditioning. It seems that the best explanation of the ill-conditioned matrix phenomenon in this case can be given as follows. If the exciting pulse with spectrum $X_{ex}(f)$ has a limited width F_m and the system is linear than the output signal with spectrum $X_{sc}(f)$ can not contain spectral components outside this band of width F_m. Moreover all signals are sampled according to equations (5) and (6), so the maximum frequency in discrete signal spectrum is 0,5 Fs = 1/(2 Ts), which is larger than F_m. So the problem is to reconstruct the vector of impulse response **h**, which is the superposition of two vectors with spectral width F_m.

The solution of equation (7) can be obtained by singular value decomposition of the excitation matrix $\mathbf{x}^{EX}$ [7]:

$$\mathbf{x}^{EX}=\mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^{T}, \quad (9)$$

where $\mathbf{U}$ and $\mathbf{V}$ are orthogonal M x M matrices, $\boldsymbol{\Sigma}$ is a diagonal matrix which elements are decreasing singular values σ_i, i = 1, …, M. The condition number Q_s of the excitation matrix $\mathbf{x}^{EX}$ can be defined by using singular values:

$$Q_{\mathbf{s}}=\frac{(\sigma_i)_{\max}}{(\sigma_i)_{\min}}, \quad (10)$$

where $(\sigma_i)_{\max}$ is the maximum singular value of the matrix $\mathbf{x}^{EX}$, $(\sigma_i)_{\min}$ is the minimum singular value of the matrix $\mathbf{x}^{EX}$. In accordance with singular value decomposition of the excitation matrix $\mathbf{x}^{EX}$ the solution of the equation (7) can be presented in the following way:

$$\mathbf{h}=\mathbf{x}^{EX^{-1}}\mathbf{x}^{SK}=\mathbf{V}\boldsymbol{\Sigma}^{-1}\mathbf{U}^{T}\mathbf{x}^{SK}, \quad (11)$$

where $\boldsymbol{\Sigma}^{-1}$ is the inverse diagonal matrix with elements $1/\sigma_i$. Equation (11) can be written as follow:

$$\mathbf{h}=\mathbf{V}\cdot\boldsymbol{\eta}, \quad (12)$$

where $\boldsymbol{\eta}$ is a vector with elements defined in the following way:

$$\eta_i = \frac{1}{\sigma_i}\left(\mathbf{U}^T\,\mathbf{x}^{SK}\right)_i, \qquad (13)$$

where $\left(\mathbf{U}^T\,\mathbf{x}^{SK}\right)_i$ is i-th element of the matrix $\mathbf{U}^T\,\mathbf{x}^{SK}$. If the matrix $\mathbf{x}^{\mathrm{EX}}$ is ill-conditioned, then small singular values σ_i will lead to the increase of the corresponding elements of the vector $\boldsymbol{\eta}$. It will cause the enlargment of the contribution to the impulse response **h** of the columns of **V** with matching indexes [7]. The physical meaning of these matrix **V** columns is that they represent discrete time signals with spectral components exceeding the limited spectral width F_m of the excitation signal. Therefore these matrix **V** columns cannot be used through the estimation of impulse response $\hat{h}(t)$.

These properties of matrix **V** columns help to determine the singular values σ_i, ($i \geq$ K, where K $\leq$ M) which can be set equal to zero. The resulting approximation of impulse response $\hat{h}(t)$ will satisfy the solution of equation (7) in the total least square meaning [8].

After obtaining the system impulse response in time-domain we proceed with the estimation of the poles using Matrix Pencil Method [6]. It is important to determine the order of the model or the number of poles, recovered from the impulse response. We have chosen the following criterion for this. We calculate the difference between initial and recovered impulse responses and evaluate the Fourier transform of this difference. If the energy of the difference spectrum in the working bandwidth of the electromagnetic structure is sufficiently low, then we have reached the order of the model.

III. The TLM Simulation of Complex Electromagnetic Structures

The TLM method is a time-space discretizing technique for the numerical analysis of electromagnetic problems [2]. The technique is based on the subdivision of the space domain into cells upon the surfaces of which the electric and magnetic fields are sampled in the two orthogonal polarizations at each cell face. The TLM cells are modelled by twelve-ports. The discretized electromagnetic structure is modelled by a mesh of transmission lines with the twelve-ports in the nodes, and the discretized electromagnetic field is represented by impulsive waves propagating on the transmission lines and being scattered at the nodes of the mesh. Given with ${}_k\boldsymbol{a}_{l,m,n}$ the 12x1 vector of the wave amplitudes incident at the cell defined by the index (l, m, n), at the k^{-th} time-step, the 12x1 vector of wave amplitudes scattered at the next time-step ${}_{k+1}\boldsymbol{b}_{l,m,n}$, is given by:

$${}_{k+1}\boldsymbol{b}_{l,m,n} = \boldsymbol{S}_k\,\boldsymbol{a}_{l,m,n}, \qquad (14)$$

where $\boldsymbol{S}$ is the scattering matrix defined as

$$\boldsymbol{S} = \begin{bmatrix} 0 & \mathbf{S}_0 & \mathbf{S}_0^T \\ \mathbf{S}_0^T & 0 & \mathbf{S}_0 \\ \mathbf{S}_0 & \mathbf{S}_0^T & 0 \end{bmatrix}, \qquad \mathbf{S}_0 = \begin{bmatrix} 0 & 0 & 0.5 & -0.5 \\ 0 & 0 & -0.5 & 0.5 \\ 0.5 & 0.5 & 0 & 0 \\ 0.5 & 0.5 & 0 & 0 \end{bmatrix}.$$

The new update of the $k+1$ incident wave amplitude is done by the hermitian unitary connection matrix $\boldsymbol{\Gamma}$ which applies as follows:

$${}_{k+1}\boldsymbol{b}_{l,m,n} = \tilde{\boldsymbol{A}}_k\,\boldsymbol{a}_{l,m,n}. \qquad (15)$$

Reiterating (1) and (2) for the entire spatial domain and over a defined observation time window, the full-wave solution of the given electromagnetic problem is obtained. In the present work the TLM has been applied to three different structures, a rectangular patch antenna, a microstrip stop-band filter on teflon substrate [3], and a broadband LTCC antenna for local multipoint distribution service (LMDS) applications between 27.5 and 29.25 GHz. This latter presents a more challenging modelling requirements since it is a multilayered three dimensional structures with high aspect ratio. The structure of the rectangular patch antenna is depicted in Fig.1.

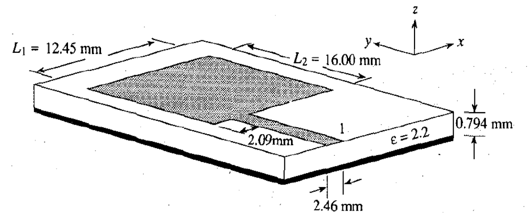

Fig. 1. Rectangular patch antenna geometry

IV. Results of Digital Modelling

Here we present the results of TLM simulations for well known patch antenna [3]. The parameters of TLM model are following: number of cells = 8745000, sampling interval = 0.2179 ps, time steps = 30000, simulation time = 1 h 11 min. The evaluated impulse response is shown in Fig. 2.

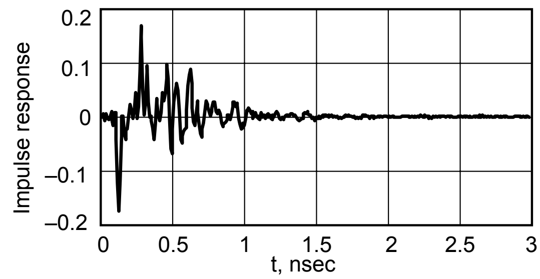

Fig. 2. Impulse response $h(t)$ of patch antenna

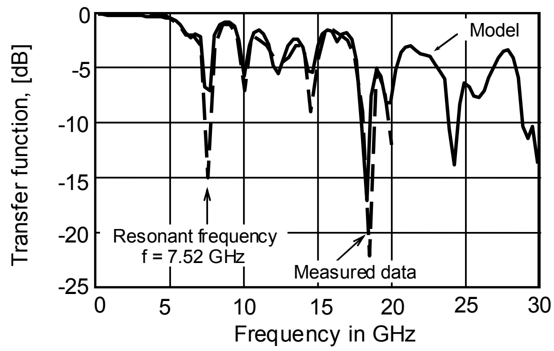

Fig. 3. Transfer function of patch antenna

The resulting transfer function corresponding to this impulse response, obtained after the decimation of initial impulse response by 60 times, is presented in Fig. 3. It is clearly seen, that the maximum frequency of the transfer function is slightly more then 30 GHz.

The positions of the estimated poles on Z-plane are shown in Fig. 4. The order of the MPM model was chosen equal to 35 in accordance with the criteria presented above. It is seen, that the positions of poles are corresponding to the frequency structure of transfer function.

The illustration of the order choice criteria is given by evaluating the difference between estimated impulse response $\hat{h}(t)$ (Fig. 2) and reconstructed impulse response $h'(t)$ according to the extracted poles (Fig. 4), which is shown in Fig. 5. You can see that the time-domain differential signal is more then order of magnitude smaller then initial impulse response.

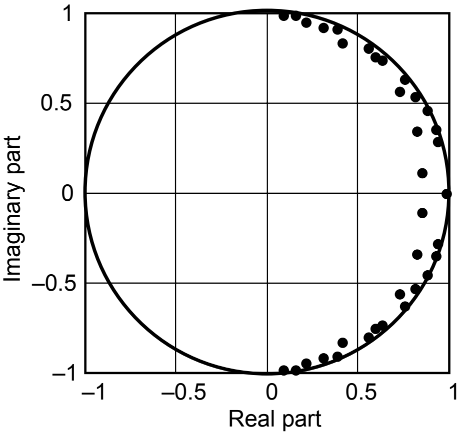

Fig. 4. Poles diagram on a complex Z-plane. The order of model $N = 35$

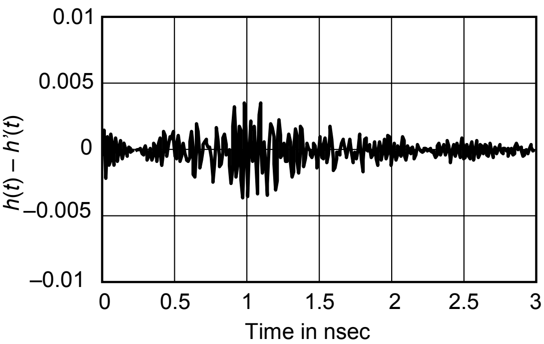

Fig. 5. The difference between recovered and initial impulse response

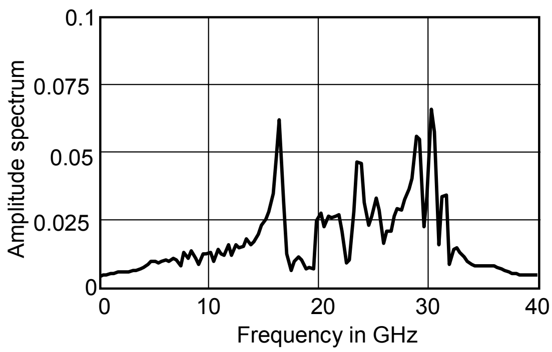

Fig. 6. The amplitude spectrum of difference between recovered signal and impulse response

One more interesting and very important thing is that the amplitude spectrum of the differential signal (Fig. 6) is concentrated outside the investigated bandwidth of the patch antenna, which is up to 20 GHz.

IV. Conclusions

We have presented a method for the numerical computation of the ultra wideband transfer function or impulse response for network representation of complex three-dimensional electromagnetic structures.

The criteria for determining the order of the MPM and the number of decimation for the simulated impulse response of the electromagnetic structures are discussed.

The presented method of network-oriented modelling, complexity reduction and system identification techniques can be combined and have the potential to reduce the computation time for the modelling of the electromagnetic structures by orders of magnitude. Furthermore these methods can help the generation of compact models of electromagnetic structures, which can be embedded in more complicated systems.

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