# PARAMETER ESTIMATION OF THE RESONANT MODEL IN PASSIVE AND ACTIVE RADAR SYSTEMS BY USING THIRD-ORDER STATISTICS

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*Abstract* — **The method based on third-order statistics (TOS) for the estimation of exponentially damped harmonic signal parameters in presence of additive Gaussian band-limited noise is presented. Contrary to the conventional approaches using date samples or correlation functions samples, the TOS method takes the advantages of cumulants and can improve the poles parameters estimation. The theory and numeral simulation results are given to demonstrate the performance of the TOS method, the increasing accuracy of complex poles estimation and the redaction of the noise threshold that can provide the range extension of active and passive radar systems.**

## I. INTRODUCTION

It is known, that various physical objects such as automobiles, planes etc. are radiant by specific wide-band electromagnetic radiation, which makes it possible to use them for objects identification. From this point of view the most informative ones are the natural electromagnetic radiations in the resonant frequency band appropriate to geometric forms and sizes of the objects. For the objects with characteristic sizes from 10 cm up to 10 m the resonant band of frequencies is from one up to hundreds of MHz.

Sources of such natural electromagnetic radiations are the physical processes accompanying with functioning of objects (electrostatic discharges, processes in combustion engines, excitation from torches of rockets etc.) or probe signals of active radar stations, if the spectrum of these signals superimposes the band of objects' resonant frequencies. The responses excited by these processes have damping oscillating character which frequencies and damping factors are determined by form and size of the object.

In accordance with singularity expansion method [1] the mathematical model of the late-time part of this target response can be decomposed into a finite sum of damped sinusoids and so the natural electromagnetic radiation of objects can be described by the resonant model. The model consists of parameters of two types: dependent on an energizing signal (polarization, form, direction of arrival) and invariant to it – the natural complex resonant frequencies of targets [2]. So the natural late-time portion of the response can be used for the aspect-independent active and passive radar target discrimination.

A lot of parametrical and non-parametrical methods of parameters estimation of resonant model can be applied to analyze the late-time natural part of the target radiation. Our study of different methods determined their noise thresholds, accessible accuracy and approximateness to Cramer-Rao bound.

Recently in the scientific literature the increasing attention is given to use higher order statistics in digital signals processing. The motivation of using cumulants is threefold [3]:

- the cumulants of Gaussian processes of order greater than two are zero and so they can be used to suppress noise under certain conditions;
- the cumulants of non-Gaussian processes include higher order statistical information about the signal;
- cumulants are phase sensitive statistics.

The main purpose of this paper is investigation and development of the TOS method for parameter estimation of resonant model in presence of Gaussian band-limited noise and the comparison of this method with traditional methods. The described in this article methods may be used for the active and passive radar target discrimination.

## II. RESONANT MODEL IN ACTIVE AND PASSIVE RADAR SYSTEMS

The response of a conducting radar target to a band-limited excitation can be represented in the late-time as a model consisted of a finite sum of damped sinusoids [2]. Such a model will allow us to estimate effectiveness and accuracy of different digital algorithms. It can be written as:

$$y_k = x_k + n_k = \sum_{t=1}^{M} |b_t| \exp\left[(\alpha_t + j\omega_t)k + j\varphi_t\right] + n_k, \quad (1)$$

where $k = 0, 1, ..., N-1$ are numbers of samples of signal $y_k$; $M$ is the number of dominant resonances induced by an exciting field; $n_k$ are samples of additive Gaussian band-limited noise; $|b_t|$, $\varphi_t$ are the aspect dependent amplitude and phase of $t$-th target mode; $\alpha_t$, $\omega_t$, are the aspect independent damping factor and natural frequency of $t$-th target mode. Note, that $z_t = \exp(\alpha_t + j\omega_t)$, $b_t = |b_t|\exp(j\varphi_t)$ are complex conjugate pairs since the field $y_k$ is real.

The parameters of signal $x_k$ have to be agreed with the real sizes of radar targets. The model of signal will be represented as a sum of three damped sinusoids. Based on results of experiments, subscribed in [4], we have assumed that the main resonant frequencies of the object are $f_1$ = 6 MHz, $f_2$ = 25 MHz and $f_3$ = 100 MHz. These frequencies correspond to geometric sizes of such objects as a automobile, a small plane, etc. As poles of resonant model have low Q-factors, we assumed that the magnitudes of Q-factors at the frequencies $f_1$, $f_2$, $f_3$ are $Q_1$ = 2, $Q_2$ = 2.25, $Q_3$ = 2.5 accordingly. The amplitudes of modes were chosen as b1: b2: b3 = 0.2: 0.4: 0.9.

The model of an additive noise $n_k$ has to correspond to conditions of real experiment, therefore we assumed that the Gaussian white noise with an infinite band is restricted at 150 MHz. The signal-to-noise ratio (SNR) for additive Gaussian band-limited noise was calculated according to the formula:

$$\mathrm{SNR} = 10 \cdot \lg\left(\frac{\mathrm{M}\left(x_k^2\right)}{\mathrm{M}\left(n_k^2\right)}\right), \tag{2}$$

where M(•) denotes the mean value. The realization of the signal with the SNR = 30 dB is depicted in Fig. 1.

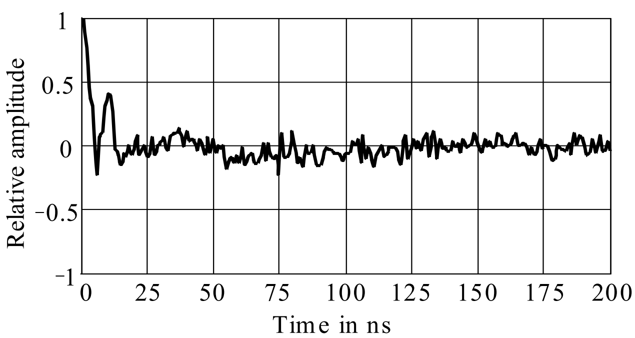


Fig. 1. The realization of the signal with the signal-to-noise ratio SNR = 30 dB

Thus the mathematical model of a signal, appropriated to the natural electromagnetic radiation of the physical object, was chosen.

## III. PRONY'S METHOD

Prony's method is one of the methods to determine the parameters of a linear combination of exponential functions. The essence of this method consists in adjustment of an exponential model to the measured data [5].

Prony's method has three main stages. On the first stage parameters of a linear prediction $a_t$ with the help of autoregressive algorithm are determined

$$\mathbf{R}\begin{bmatrix} 1 \\ a_1 \\ \vdots \\ a_M \end{bmatrix} = \begin{bmatrix} \rho \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \tag{3}$$

where $\mathbf{R}$ is an autocorrelation matrix $(M + 1) \times (M + 1)$; $a_t$, $t = 1, 2, \ldots, M$ are coefficients of linear forward-prediction and $\rho$ is an average error of a linear forward-prediction.

On the second stage from the received vector $\mathbf{a}$ a polynomial is formed

$$\Psi(z) = \sum_{t=0}^{M} a_t z^{M-t} \tag{4}$$

and its roots $z_t$ are defined. The estimation of the damping factors $\alpha_t$ and natural frequencies $\omega_t$ are determined as

$$\alpha_t = \frac{\ln|z_t|}{T}, \qquad \omega_t = \frac{1}{T} \cdot \mathrm{arctg}\left[\frac{\mathrm{Im}\{z_t\}}{\mathrm{Re}\{z_t\}}\right], \tag{5}$$

where $T$ is a period of discretization of a signal.

On the third stage the system of linear equations for the calculation of magnitudes and initial phases is determined. Studying classical versions of this method has resulted in its improvement and generalized versions and understanding their noise characteristic features.

## IV. THIRD-ORDER STATISTICS METHOD

The third-order statistics method also consists of three main stages [3]. First of all the third-order correlation sequence (third-order cumulants) is calculated

$$R(m, n) = \frac{1}{N}\sum_{k=0}^{N-1} y_k \cdot y_{k+m}^{*} \cdot y_{k+n}^{*}, \tag{6}$$

where "*" denotes complex conjugate.

On the second stage the system of linear equations should be constructed

$$\mathbf{R}_y \cdot \mathbf{a}' = -\mathbf{r}_y, \tag{7}$$

where

$$\mathbf{a}' = \left(a_1 \; a_2 \; \cdots \; a_M\right)^T \tag{8}$$

is an autoregressive coefficient vector of the resonant model of the signal $x_k$ and $T$ denotes the transposition. The most informative 1-D slice of the third-order cumulant $R(m,n)$ (Fig. 2) should be found to form $\mathbf{R}_y$ and $\mathbf{r}_y$. Two different slices which can be used are depicted in Fig. 3 and Fig. 4. So from $R(m,m)$ we can determine:

$$\mathbf{R}_y = \begin{pmatrix} R(2, 2) & \cdots & R(M+1, M+1) \\ R(3, 3) & \cdots & R(M+2, M+2) \\ \vdots & \ddots & \vdots \\ R(M+1, M+1) & \cdots & R(2M, 2M) \end{pmatrix}, \tag{9}$$

$$\mathbf{r}_y = \left(R(1, 1)\, R(2, 2) \cdots R(M, M)\right)^T. \tag{10}$$

The minimum norm solution of the linear system (7) can be, for example, given as [3]:

$$\mathbf{a}' = \left(\left(\mathbf{R}_y\right)^H \cdot \mathbf{R}_y\right)^{-1} \cdot \left(\mathbf{R}_y\right)^H \cdot \mathbf{r}_y, \quad (11)$$

where $H$ denotes transposition and conjugation.

On the third stage of the TOS method the polynomial (4) using coefficients $\mathbf{a}'$ are formed. Then the roots of this polynomial are determined and the estimations of resonant frequencies and damping factors of a resonant model are found according to formula (5).

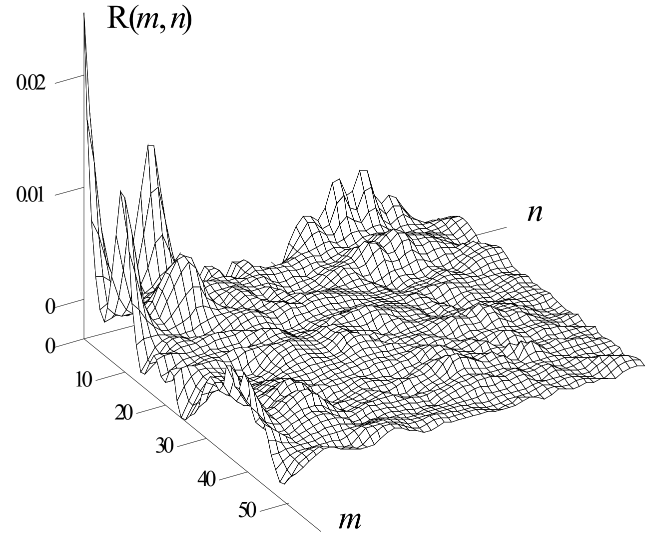


Fig. 2. 2-D sequence of the third-order cumulants of the resonant model

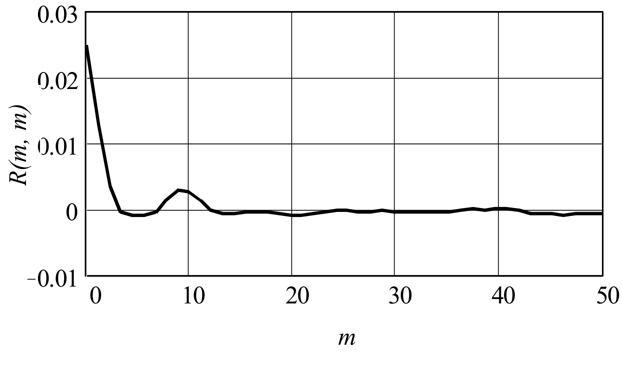


Fig. 3. 1-D slice $R(m, m)$ of the third-order cumulants of the resonant model

Note, that according to the simulations the slice R(m,0) for the forming (9) in case of used resonant model can be successful used.

## V. RESULTS OF DIGITAL MODELLING

As it was investigated [6], traditional methods of parameters estimation of the resonant model, such as Prony's method, are not able to estimate poles of resonant model with a appropriate accuracy if the SNR less as 20dB.

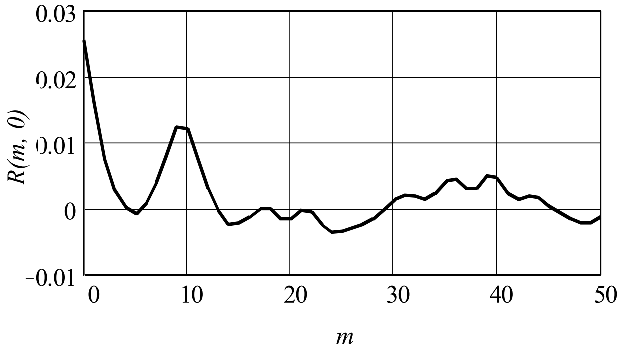


Fig. 4. 1-D slice $R(m, 0)$ of the third-order cumulants of the resonant model

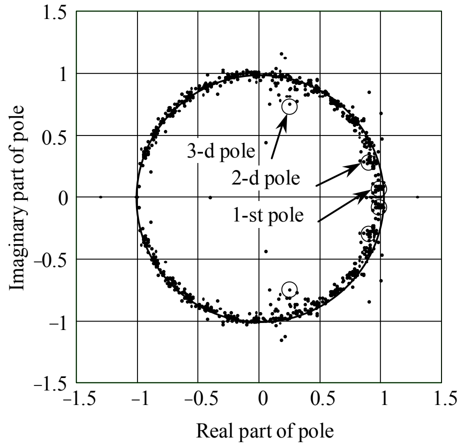


Fig. 5. Estimates of poles for 50 independent runs using Prony's method, SNR = 5 dB

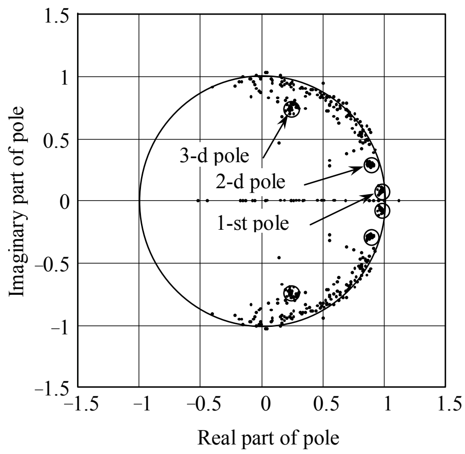


Fig. 6. Estimates of poles for 50 independent runs using TOS method, SNR = 5 dB

So we have carried out a comparison of this method with the TOS method. The poles of the resonant model were estimated by algorithms described in sections III and

IV. The results of poles estimation by TOS and Prony's methods for 50 independent realizations of Gaussian band-limited noise with SNR = 5 dB are depicted in Fig. 5 and Fig. 6. The true signal poles locations are depicted as circles, the estimations - as points. It should be note, that poles defined by TOS, in contrast to Prony's method, have a significantly less variance and concentrate around their true values.

The quantitative comparison of effectiveness of the above considered methods was made by using the variance of poles:

$$D = \sum_{i=1}^{K} \left( \frac{\left| z_{t,i} - z_t \right|}{\alpha_t} \right)^2 \Big/ K , \qquad (12)$$

where $z_t$ is $t$-th pole of the signal; $z_{t,i}$ is estimation of $t$-th pole for $i$-th trial run of $y_k$ (1); $\alpha_i$ is damping factor of $t$-th pole; $K$ is a number of independent trial runs. Each sample of variance was computed by $K$ = 50 trial runs. Independent Gaussian band-limited noise sequence each run was perturbed with the $x_k$. The SNR was estimated in accordance with (2). The simulation results for the TOS method are depicted in Fig. 7.

The comparison of the two methods by estimation of variation for the first pole is depicted in Fig. 8. We can conclude, that by using TOS method the noise threshold of poles estimation can be significantly shifted and estimation accuracy can be improved on 3-7dB. If a threshold level of the accuracy is defined, for example, by variance $D$ = -5 dB, Prony's method gives satisfactoral results for the low-frequency pole only at the SNR = 16 dB, while the TOS method – at SNR = 5 dB.

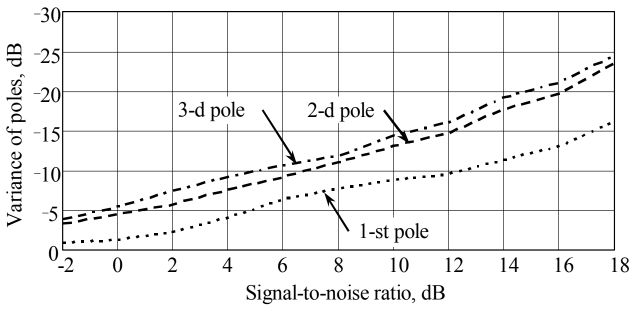


Fig. 7. Variance of the poles estimated by TOS method

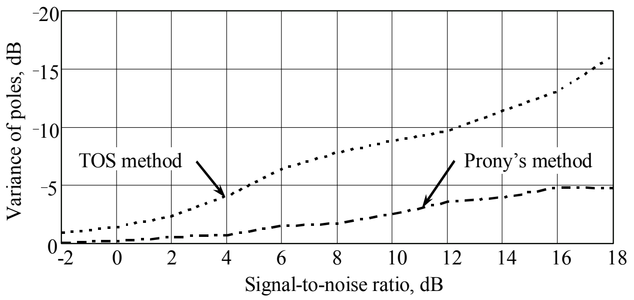


Fig. 8. Variance of the 1-st pole estimated by TOS and Prony's methods

## VI. CONCLUSIONS

The third-order statistics were used for the purpose of parameter estimation of the targets resonant models. The model consisting of three damped sinusoids and additive Gaussian band-limited noise was chosen. We determined the most informative 1-D slice of the 2-D sequence of third-order cumulants and defined the parameters of the model. Comparison estimation the TOS method with the traditional Prony's method proved its high accuracy of the poles determination. The third-order statistics can be used as an effective tool for the determination of resonant model parameters as well in the passive and active radar systems as in other different applications of signal processing.

The results of digital modeling shows that the using of TOS method for parameter estimation of the resonant model leads to accuracy increase of the estimation of poles on 3-7 dB in comparison with traditional algorithms, that can provide the increasing range of active and passive radar stations up to three times.

## VII. REFERENCES

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